(4)

TECHNICAL REPORT ON NONLINEAR WAVES

OFFICE OF NAVAL RESEARCH GRANT #N00014-88-K-0447

BY

MARK J. ABLOWITZ

PRINCIPAL INVESTIGATOR

AND

ATHANASSIOS S. FOKAS

CO-PRINCIPAL INVESTIGATOR

DEPARTMENT OF MATHEMATICS

AND

COMPUTER SCIENCE

CLARKSON UNIVERSITY

POTSDAM, NY 13676

JUNE 15, 1989

Overview

The current research funding has been used to support the research activities of Professors Mark J. Ablowitz and Athanassios S. Fokas and associates. The investigators have been working in the general area of nonlinear wave propagation for over twenty years. The main focus of this work is the understanding of the nonlinear phenomena involved with the wave propagation arising in physical problems. The work has application to numerous areas of physics, engineering and mathematics. Applications include fluid dynamics, waves in stratified fluids, surface gravity waves and wave excitation phenomena related to moving pressure distributions; numerical approximation and computation; nonlinear optics; and plasma physics. Moreover the study of solutions to some of the underlying nonlinear evolution equations has led naturally to the investigation and new results in the separate but closely related field of inverse scattering. Developments to both one and multidimensional inverse problems have been made.

During the period of support of this grant there has been substantial activity. The investigators have over 30 research papers published and more than 10 papers accepted for publication; with still a number of papers ready to be submitted. There are a number of research directions and problems we are pursuing. These include the following.

(i) Development of solutions to multidimensional nonlinear evolution equations of physical significance. Prototypes are the so-called Kadomtsev-Petviashvilli and Davey-Stewartson equations. These equations arise in a variety of physical instances such as water waves, stratified fluid flow, plasma waves etc. We have found some new and important results. The nature of the boundary value problems and solutions of the equations in the so-called strong coupling limit have recently been uncovered. The role of the boundary conditions and a number of the essential differenes between one and two (spatial) dimensional problems have been clarified. The role

of the boundary conditions is important in the understanding of why highly localized multidmensional soliton solutions exist for one of the Davey-Stewartson (D-S) equations (i.e. DS-I but not DS-II). These multi-dimensional soliton solutions have been found by a novel application of the Inverse Scattering Transform. Specifically the soliton solutions correspond to a nontrivial mean field contribution at infinity. Considering the asymptotic description by which the D-S equations are derived, this means that energy must be inserted to the physical field near the boundary. Moreover rigorous theorems regarding the existence of solutions have been obtained under certain assumptions.

(ii) Solutions of discrete nonlinear evolution equations and numerical computation. In our studies we have found the following surprising situation. Namely associated with the integrable nonlinear Schrodinger equations are standard numerical schemes which exhibit at intermediate levels of mesh refinement a weak form of temporal chaos. On the other hand not all approximations exhibit this instability. In fact, difference schemes developed by Inverse Scattering Transform (IST) methods do not exhibit this spurious chaos. All schemes agree when the mesh is sufficiently refined.

(iii) Inverse problems associated with multidimensional problems. A key element in this work is the DBAR method developed by the principal investigators and their colleagues a few years ago. The method has been extended from the study of two dimensional inverse problems to n dimensions ($n>2$). In principal the method can be applied to inverse problems arising in geophysics and acoustics. Rigorous theorems have been obtained for a number of interesting multidimensional problems.

(iv) Cellular Automata and solitons. The principal investigators and their associates have been studying a class of cellular automata which admit solitons as special solutions. Interesting new classes of solutions have been found. Recently these automata have been generalized to two and three dimensions and we have found instances of soliton interaction. These systems are not reversible, which is quite a novel and interesting aspect.

(v) Painleve equations. in our study of Painleve equations we have developed a method to linearize these classical nonlinear ordinary differential equations. The linearization is provided by a system of Riemann-Hilbert boundary value problems which can be related to a system of linear integral equations. A rigorous theory is under development.

(vi) Moving pressure distributions in surface gravity waves and semi-infinite boundary value problems. A general theory to study semi-infinite boundary value problems has been formulated. Solutions to these systems are under study and a connection between certain boundary value problems on the semi-infinite line and forced nonlinear wave equations has been obtained.

(vii) Solutions to a class of nonlinear singular integro-differential equations have been developed. These include the well known Benjamin-Ono, Intermediate Long Wave and Sine-Hilbert equations which arise in stratified fluid dynamics. We have recently been studying multidimensional nonlinear singular integro-differential equations and have a number of interesting results.

(viii) Analysis and development of the symmetries, Hamiltonian structures and recursion operators for two spatial dimensional problems has been constructed. A general theoretical framework has been developed.

The broad attack is to understand the behavior of coherent structures and solutions to nonlinear equations arising in physical problems. The results obtained and wide ranging interest by scientists and engineers

in our work have motivated many of the studies. In what follows is a list of recent publications.

PUBLICATIONS

1. Solutions of Multidimensional Extensions of the Anti-Self-Dual Yang-Mills Equations, M.J. Ablowitz, D.G. Costa and K. Tenenblat, Stud. Appl. Math., 77:37-46, 1987.

2. Note on Solutions to a Class of Nonlinear Singular Integro-differential Equations, M.J. Ablowitz, A.S. Fokas and M.D. Kruskal, Phys. Lett. A., Vol. 120, 5 pp. 215-218, 1987.

3. Exactly Solvable Multidimensional Nonlinear Equations and Inverse Scattering, M.J. Ablowitz, Proceedings of Nonlinear Evolution Equations, Solitons and the IST, Oberwolfach, Germany 1986. Ed. by M.J. Ablowitz, M.D. Kruskal and B. Fuchssteiner, 20-32, World Scientific Publ. Co., 1987.

4. Topics Associated with Nonlinear Evolution Equations and Inverse Scattering in Multidimensions, M.J. Ablowitz, Ed. by M. Lakshmanan, Proceedings of "Solitons", Winter School, Tiruchirapalli, India, 44-66, 1987, Springer-Verlag, 1988.

5. Numerical Simulation of the Modified Korteweg-deVries Equation, Thiab R. Taha and M.J. Ablowitz, Adv. Comp. Methods for Partial Differential Equations IV. Ed. by R. Vichnevetsky and R.S. Stepleman 1987.

6. Hodograph Transformations of Linearizable Partial Differential Equations, A.S. Fokas, P.A. Clarkson and M.J. Ablowitz, INS#78, April 1987, to be published in SIAM Jrnl. on Appl. Math.

7. On the Initial Value Problem for a Class of Nonlinear Integral Evolution Equations Including the Sine-Hilbert Equation, P.M. Santini, M.J. Ablowitz and A.S. Fokas, J. Math. Phys. 28(10) October 1987.

8. Davey-Stewartson I - A Quantum 2+1 Dimensional Integrable System, C.L. Schultz, M.J. Ablowitz and D. Bar Yaacov, Phys. Rev. Lett. 59, #25, 2825-2828, 1987.

9. Inverse Scattering Transform Numerical Schemes for Nonlinear Evolution Equations of Physical Interest, T.R. Taha and M.J. Ablowitz, Num. Approx. of Partial Differential Equations. Ed. E.L. Ortiz, North-Holland, 1987.

10. Classical Poisson Bracket Relations and Quantum Commutation Relations for Davey-Stewartson, C.L. Schultz, M.J. Ablowitz, D. Bar Yaacov. Proceedings of the IVth Workshop on Nonlinear Evolution Equations and Dynamical Systems, Balaruc-les Bains, France, June, 1987. Ed. J.P. Leon, 437-448, World Scientific 1988.

11. A Rule for Fast Computation and Analysis of Soliton Automata, T.S. Papatheodorou, M.J. Ablowitz and Y.G. Saridakis, Stud. in Appl. Math. 79, 1988.

12. Analytical and Numerical Aspects of Certain Nonlinear Evolution Equations IV, Numerical, Modified Korteweg-deVries Equation, T.R. Taha and M.J. Ablowitz, J. Comp. Physics, Vol. 77, No. 2, August 1988.

13. A Method of Linearization for Painlevé Equations: Painlevé IV, V, A.S. Fokas, U. Mugan and M.J. Ablowitz, Physica D 30, 1988.

14. Action-Angle Variables and Trace Formula For D-Bar Limit Case Of Davey-Stewartson I, C.L. Schultz and M.J. Ablowitz, Phys. Lett. A., Vol. 135, No. 8,9, March 1989.

15. Strong Coupling Limit of Certain Multidimensional Nonlinear Wave Equations, M.J. Ablowitz and C.L. Schultz, Stud. in Appl. Math. 80:229-238, June 1989.

16. Numerically Induced Chaos in the Nonlinear Schrödinger Equation, B. Herbst and M.J. Ablowitz, Phys. Rev. Lett., Vol. 61, No. 18, 1989.

17. Interaction of Simple Particles in Soliton Cellular Automata, A.S. Fokas, E.P. Papadopoulou, Y.G. Saridakis and M.J. Ablowitz, INS#97, June 1988, to be published Stud. in Appl. Math.

18. Forced Nonlinear Evolution Equations and the Inverse Scattering Transform, A.S. Fokas and M.J. Ablowitz, Stud. in Appl. Math., Vol. LXXX No. 3, 253-272, June 1989.

19. Exact Solutions of Nonlinear Evolution Equation in the Strong Coupling Limit, C.L. Schultz and M.J. Ablowitz, to be published Proceedings for International Workshop on Nonlinear Evolution Equations: Integrability and Spectral Methods, Como, Italy, July 4-15, 1988.

20. Nonlinear Evolution Equations, Solitons and Cellular Automata, INS#111, to be published Proceedings for Singular Behavior and Nonlinear Dynamics held in Samos, Greece, August 1988.

21. Painlevé Equations and the Inverse Scattering and Inverse Monodromy Transforms, M.J. Ablowitz, INS#105, to be published, Proceedings on Solitons in Physics and Mathematics, Institute of Math and Its Applications, IMA, Minneapolis, Minnesota, September 1988.

22. Nonlinear Evolution Equations, Inverse Scattering and Cellular Automata, M.J. Ablowitz, INS#114, to be published, Proceedings Solitons in Nonlinear Optics and Plasma Physics, Institute of Mathematics and Its Applications, IMA, Minneapolis, Minnesota, November 1988.

23. Solitons, Inverse Problems and Nonlinear Equations, INS#118, M.J. Ablowitz, to be published Jrnl. of Comp. and Appl. Math., February 1989.

24. On Numerical Chaos in the Nonlinear Schrödinger Equations, B. Herbst and M.J. Ablowitz, to appear Proceedings of Workshop on Complete Integrability, Orleon, France, INS#120, January 1989.

25. Rounding Error and the Loss of Spatial Symmetry Associated with Discretizations of the Nonlinear Schrödinger Equation, B.M. Herbst and M.J. Ablowitz, INS#122, April 1989.

26. Nonlinear Evolution Equations, Solitons, Strange Attractors and Cellular Automata, M.J.Ablowitz, B.M. Herbst and J.M. Keiser, INS#123, April 1989.

27. Recursion Operators and Bi-Hamiltonian Structures in Multidimensions I, P.M. Santini and A.S. Fokas, Comm. Math. Phys. 115, 375-419 (1988).

28. Recursion Operators and Bi-Hamiltonian Structures in Multidimensions II, A.S. Fokas and P. M. Santini, Commun. in Math. Phys. 116, 449-474 (1988).

29. Symmetries and Integrability, A.S. Fokas, Stud. Appl. Math. 77, 253-299 (1987).

30. Aspects of Integrability in One and Several Dimensions, Proceedings of Nonlinear Evolutin Equations, Solitons and the IST, A.S. Fokas, Oberwolfach, Germany, 1986, Ed. by M.J. Ablowitz, M.D.Kruskal and B. Fuchssteiner, World Scientific (1987).

31. Integrable Equations in Multidimensions (2+1) are Bi-Hamiltonian Systems: Proceedings of "Solitons", Winter School, A.S. Fokas and P.M. Santini, Tiruchirapalli, India, ed. by M. Lakshmanan, Springer-Verlag (1988).

32. Inverse Problems and a Unified Approach to Integrability in 1, 1+1, and 2+1 Dimensions, A.S. Fokas, V.Papageorgiou, ed. by M. Lakshmanan, Proceedings of "Solitons", Winter School, Tiruchirapalli, India, Springer-Verlag 1988.

33. Inverse Scattering Transform on the Half-Line: The Nonlinear Analogue of the Sine Transform Inverse Problems, A.S. Fokas, Montpellier, France, November 1986. Ed. by P. Sabatier, Academic Press (1988).

34. Symmetries and Bi-Hamiltonian Structures of 2+1 Dimensional Systems, P.M. Santini and A.S. Fokas, Proceedings of Nonlinear Evolution Equations, Solitons and the IST, Oberwolfach, Germany, 1986, ed. by M.J. Ablowitz, M.D. Kruskal and B. Fuchssteiner, World Scientific (1987).

35. Bi-Hamiltonian Formulation of the Kadomtsev-Petviashvili and Benjamin-Ono Equations, A.S. Fokas and P.M. Santini, J. Math. Phys. 29 (3) 604-617 (1988).

36. An Initial Boundary Value Problem for the Nonlinear Schrodinger Equations, A.S. Fokas, Physica D March 1989.

37. Evolution Theory, Periodic Particles and Solitons, T.S. Papatheodorou and A.S. Fokas, to appear Stud. in Appl. Math., 1989.

38. The Bi-Hamiltonian formulation of the Landau-Lifshitz Equation, E. Barouch, A.S. Fokas and V. Papageorgiou, J. of Math. Phys., 29, 2628 (1988).

39. Algorithmic Construction of the Recursion Operators of Toda and Landau-Lifshitz Equations, E. Barouch, A.S. Fokas and V. Papageorgiou, RIMS Publications 650, 179-196 (1988).

40. Conservation Laws for Integrable Systems, A.S. Fokas and P.M. Santini, Proceedings of International School on Applied Mathematics: Symmetries and Nonlinear Phenomena, Bogota, Colombia, Feb. 22-27, 1988, ed. by G. Violini, D. Levi and P. Winternitz.

41. On the Time Evolution Equations of Spin Systems and Their Continuum Limit, E. Barouch, A.S. Fokas and V. Papageorgiou, Proceedings of International School on Applied Mathematics: Symmetries and Nonlinear Phenomena, Bogota, Colombia, Feb. 22-27, 1988, ed. by G. Violini, D. Levi and P. Winternitz.

42. The Bi-Hamiltonian Formulations of Integrable Evolution Equations in Multidimensions, P.M. Santini and A.S. Fokas, Proceedings of the Workshop "Nonlinear Evolution Equations and Dynamical Systems", Balaruc Les Bains, France.

43. Inverse Problems and Integrable Equations in Multidimensions, L.Y. Sung and A.S. Fokas, INS#98, Clarkson University, January 1989.

44. A Unified Approach to Recursion Operators, A.S. Fokas and P.M. Santini, INS#101, Clarkson University, September 1988, to be published, Proceedings of Workshop on Solitons in Nonlinear Optics and Plasma Physics, September 1988, IMA, Minnesota.

45. Mathematical Modeling of Chronic Myelogenous Leukemia, A.S. Fokas, J. Keller and Bayard Clarkson, December 1988.

46. Some Recent Results in Soliton Theory, A.S. Fokas, INS#104, Clarkson University, October 1988.

47. Solitons in Multidimensions, A.S. Fokas and P.M. Santini, INS#106, Clarkson University, November 1988.

48. Particles in Soliton Cellular Automata, A.S. Fokas, E. Papadopoulou and Y. Saridakis, INS#107, Clarkson University, November 1988.

49. New Manifestations of Solitons, A.S. Fokas, INS#112, Clarkson University, December 1988, to be published, Proceedings of Workshop on Solitons in Nonlinear Optics and Plasma Physics, November 7-11, 1988, IMA, Minnesota.

50. Linearized Maps for the Davey-Stewartson I Equation, A.S. Fokas, INS#116, Clarkson University, January 1989, to be published, Proceedings of Workshop on Integrable Systems and Applications at Ile d'Oleron, June 20-24, 1988, France.

51. Solitons and a Boundary Value Problem for the Davey-Stewartson I Equation, A.S. Fokas and P.M. Santini, INS#121 Clarkson University, March 1989.